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Inertial Compensation of Energy Losses
in Timing Mechanisms

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INERTIAL COMPENSATION OF

ENERGY LOSSES IN

TIMING MECHANISMS

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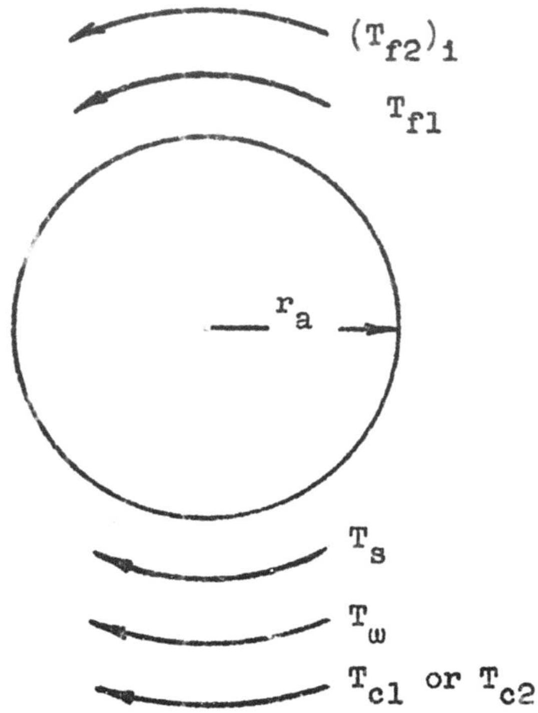
ABSTRACT

The behavior of timing mechanisms under spin conditions is affected by: 1) changes in gear train friction, 2) changes in friction between main spring leaves, 3) changes in mainspring torque. The purpose of this study has been to examine these effects in the light of the possibility of synthesizing an inertial (spin energized) compensator which will provide enough torque to balance out both spin induced and run-down torque losses.

The problem has been approached by analyzing various sources of torque loss or gain and then combining these analyses to show the pertinent design parameters. In the work that follows, all torques will be referred to the spring arbor as shown in Fig. 1.



$(T_{f2})_i$	Torque due to friction between ith and i-1 spring leaves
T_{f1}	Torque due to spin induced friction in gear train
T_s	Torque due to windup of main spring
T_ω	Torque due to spin of main spring
T_{c1}	Torque due to segmental gear compensator
T_{c2}	Torque due to attached mass compensator

Fig. 1

ANALYSIS

Gear Train Friction The loads, stress levels, and friction torques of timer-mechanism bearings have been studied in detail in [1] and [2].* For the case of the conventional timer (shafts parallel to the axis of spin) the major spin induced friction torques are generated in journal bearings. A cross-section view of such a bearing is shown in Fig. 2. A cylindrical shaft of diameter d turns in part of a toroidal journal bearing which has an inside diameter $d + c$ and meridian radius R. As a limiting case the bearing may be cylindrical, ie., $R = \infty$ and the axial clearance shown may not exist or may be replaced by some provision for withstanding a small thrust load.

If a cosine distribution of normal force in the circumferential direction is assumed, the following relations can be derived for maximum pressure and friction torque [2] for the particular shaft-gear assembly

1) $$(p_m)_k = \frac{4(P_{tr})_k}{\pi d_k}$$

2) $$(T_f)_k = \frac{2\mu_1 d_k (P_{tr})_k}{\pi}$$

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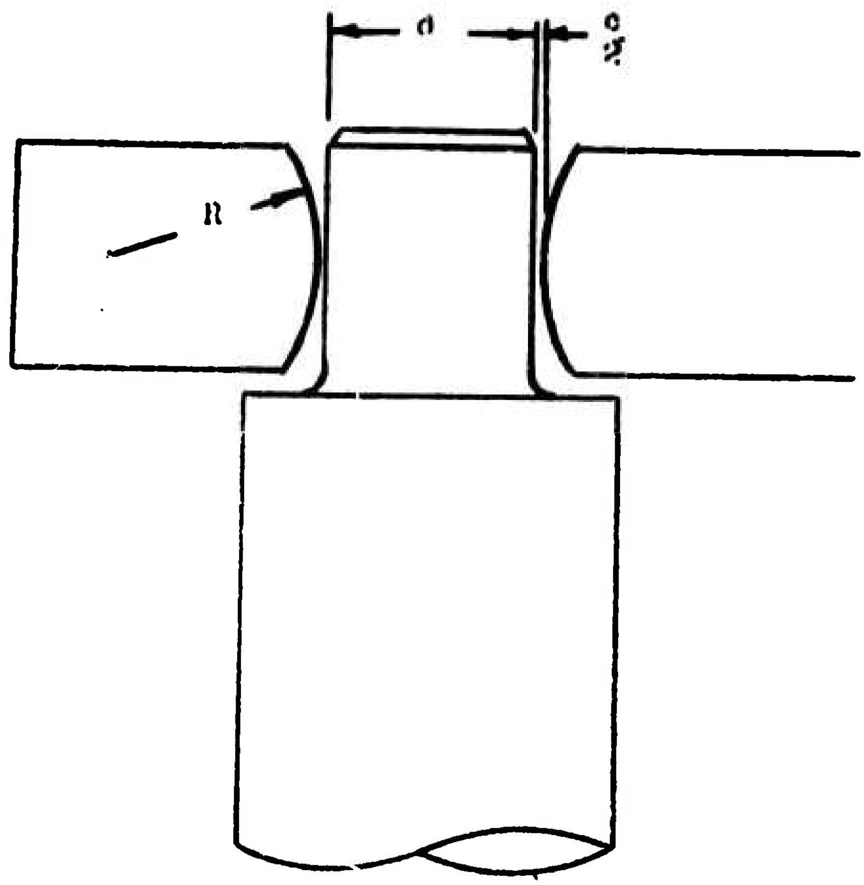
*Bracketed numbers designate references listed on page 44



SCHEMATIC OF JOURNAL BEARING

Fig. 2

where $(P_{tr})_k$ is the transverse load applied to the kth bearing, $(P_m)_k$ is the maximum force per unit length of circumference, d_k is the diameter of the kth shaft, μ_1 is the coefficient of friction, and $(T_f)_k$ is the friction torque at the kth shaft. If it is assumed that transverse forces produced by transmitted torque are negligible compared with those produced by angular acceleration, relation 2) may be written [2]

$$3) \qquad (T_f)_k = \frac{2\mu_1 \; d_k m_k e_k \omega^2}{\pi}$$

where m_k is the mass of the shaft-gear assembly, e_k is the eccentricity of the shaft, and ω is the angular velocity of the spinning spring.

The friction torque $(T_{f1})_k$ reflected to the mainspring for each shaft-gear assembly is

$$4) \qquad (T_{f1})_k = (T_f)_k \, g_k$$

where g_k is the gear ratio to the main spring. If relation 3) is used, 4) becomes

$$5) \qquad (T_{f1})_k = \frac{2\mu_1 \; d_k m_k e_k g_k \omega^2}{\pi}$$

and the total friction torque reflected to the main spring is

6) $$T_{fi} = \frac{2\mu_i}{\pi} \left[\sum_{k=1}^{u} d_k m_k e_k g_k \right] \omega^2$$

where u is the number of gear-shaft assemblies in the gear train.

<u>Mainspring Friction</u> The rotating spring is simulated by concentric sections offset as shown in Fig. 3.

A general expression for the length ℓ_i of the ith turn of the spring is

7) $$\ell_i = \pi \left[2r_a + (2i - 1)t + d_1 + d_2 + - - - + d_i) \right]$$

where t is the thickness of the spring and r_a is the radius of the arbor.

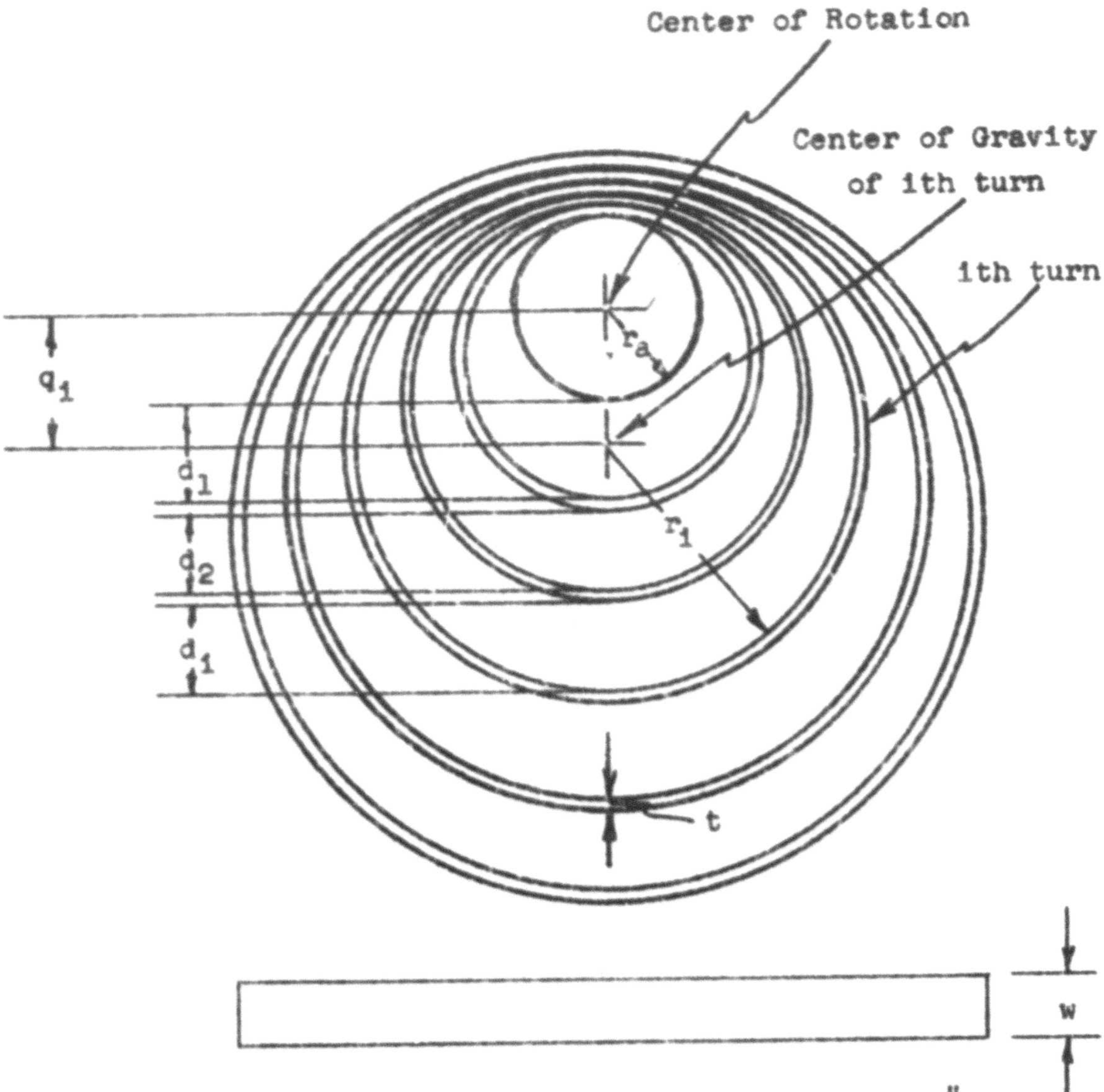
The radius of the ith turn is $\ell_i / 2\pi$ and the distance q_i from the center of rotation of the arbor to the center of gravity of the ith turn is

8) $$q_i = \frac{\ell_i}{2\pi} - (r_a + \frac{t}{2}) - it$$

Assuming a density of 500 pounds per cubic foot, the mass of the ith turn is

9) $$m_i = \ell_i \ (t) \ (w) \ (120)10^{-4} \qquad (\text{oz. sec.}^2/\text{in.})$$

where w is the width of the spring.



Example: $r = .14''$
$w = .25''$
$t = .015''$

Fig. 3

The normal force exerted on the i-1 turn by the ith turn then is

$$10)\qquad F_i = \sum_{j=i}^{n} m_j q_j \omega^2$$

Substituting in 10) from 9), 8), and 7) gives

$$11)\qquad F_i = \frac{120tw}{10^4}\left[\frac{1}{2\pi}\sum_{j=i}^{n} (\ell_j)^2 - (r_a + \frac{t}{2})\sum_{j=i}^{n} \ell_j - t\sum_{j=i}^{n} j\ell_j\right]\omega^2$$

where

$$12)\qquad \ell_j = \pi\,[2r_a + (2j - 1) + \sum_{i=1}^{j} d_i$$

The friction torque $(T_{f2})_i$ is that occurring between the ith and i-1 turn and may be written

$$13)\qquad (T_{f2})_i = [r_a + (i - 1)t]\mu_2 F_i$$

where μ_2 is the coefficient of friction.

Static run-down photographs indicate that to a good approximation the distances d_1, d_2, - - - d_i may be assumed to remain equal. These distances will, however, depend upon the number of turns of the spring. Thus from 7) when $d_1 = d_2 = - - - - = d_i = d_n$

14) $$\ell = \pi \sum_{i=1}^{n} \left[2r_a + (2i - 1)t + (d_n)_1 + (d_n)_2 + \right.$$

$$\left. - - - + (d_n)_i \right]_i$$

and

15) $$\ell = \pi \left[n(2r - t) + tn(n + 1) + d_n \frac{n(n + 1)}{2} \right]$$

and

16) $$d_n = \frac{2\,[\ell/\pi - tn^2 - 2r_a n]}{n(n + 1)}$$

Equation 11 may then be reduced to

17) $$F_1 = \frac{120tw}{10^4} \left[A \sum_{j=1}^{n} 1 + B \sum_{j=1}^{n} j + C \sum_{j=1}^{n} j^2 \right] \omega^2$$

where the coefficients A, B_1 and C are

$$A = - \pi(2r_a t - t^2)$$

$$B = \frac{\pi}{2}\left(2r_a d_n - 3td_n - 4t^2\right) \tag{18}$$

$$C = \frac{\pi}{2}\left(d_n^{\,2} + 2td_n\right)$$

Summing 17) gives

$$F_i = \frac{120tw}{10^4}\left\{ A(n + 1 - i) + \frac{B}{2}\left[n(n + 1) - i(i - 1)\right] + \frac{C}{6}\left[n(n + 1)(2n + 1) - i(i - 1)(2i - 1)\right] \right\}\omega^2 \tag{19}$$

and finally

$$(T_{f2})_i = \frac{120\ \mu_2\ tw}{10^4}\left[r_a + (i - 1)t\right] \left\{A\ (n + 1 - i) + \frac{B}{2}\left[n(n + 1) - i(i - 1)\right] + \frac{C}{6}\left[n(n + 1)(2n + 1) - i(i - 1)(2i - 1)\right]\right\} \omega^2 \qquad \text{(in.lb.)} \tag{20}$$

<u>Mainspring Torque</u> Changes in spring torque occur as a result of inertial forces acting on spring leaves during spin. These inertial forces are proportional to the square of the spin speed ω and to the distance of the spring leaves from the arbor. For a given spin speed

inertial forces become larger as the spring unwinds.

In contrast to the inertial effects which ideally increase the spring torque, static spring torque decreases as the spring unwinds. This decrease, when the spring is not spinning, is for all practical purposes proportional to angle of unwind. For the effective range of the spring, a theoretical result that agrees well with experiment is

$$21) \qquad T_s = \frac{EI}{l}(\theta_o - \theta) + D$$

where T_s is the spring torque, E is Young's modulus, I is cross-section moment of inertia of the spring, l is the length, θ is the angle of unwind of the arbor of the spring, and θ_o is the fully wound angle of the arbor. The constant D must be determined experimentally and is the torque intercept of the static run-down curve as shown in Fig. 4.

For the case of a spring having the following spiral shape [3]

$$r = R_2 - \frac{R_2 - R_1}{\Phi} \beta$$

where Φ is the total angle of the spiral, β is the an-

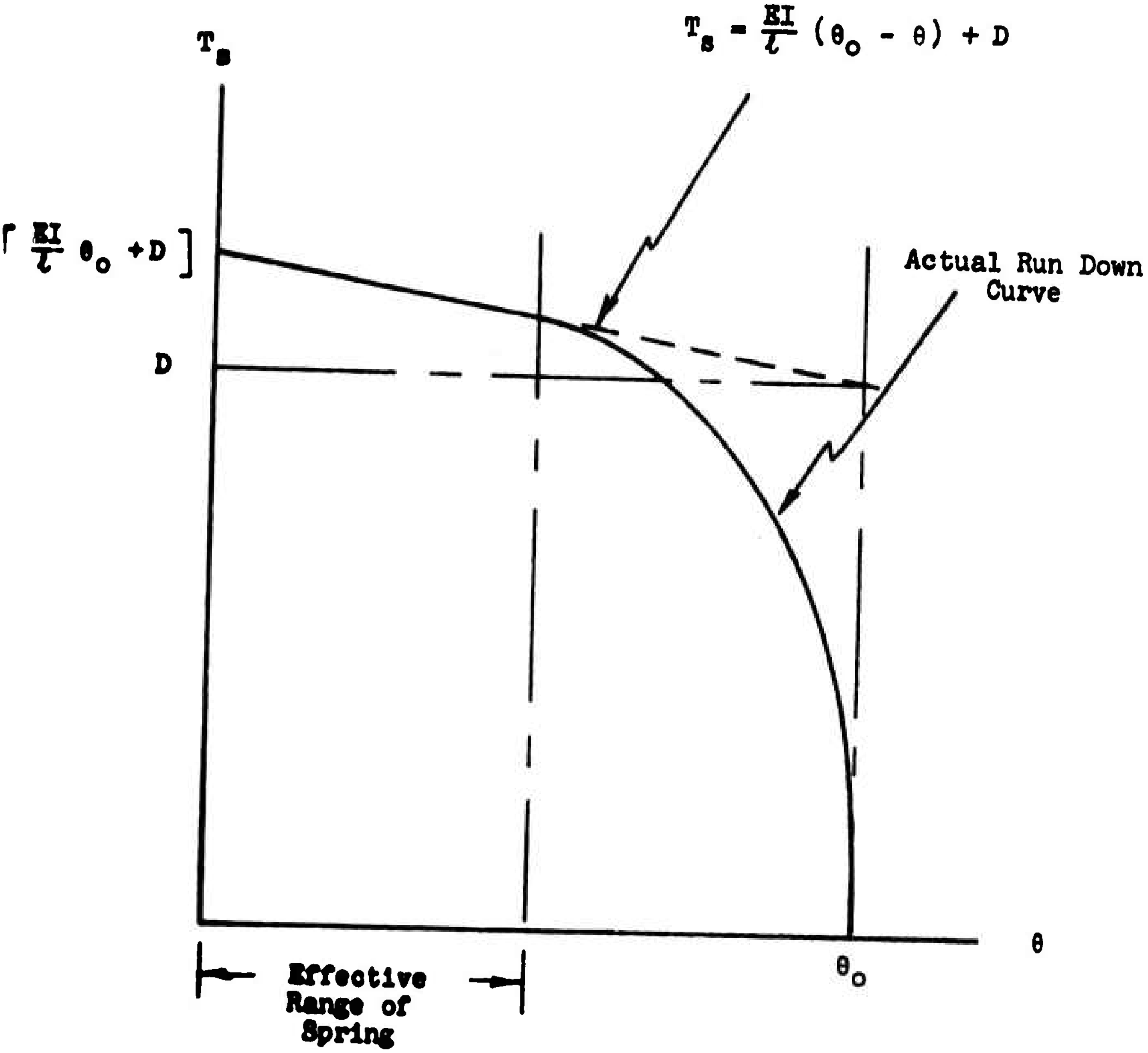

T_s
$T_s = \frac{EI}{l}(\theta_o - \theta) + D$
$\left[\frac{EI}{l}\theta_o + D\right]$
D
Actual Run Down Curve
θ
θ_o
Effective Range of Spring

Fig. 4

gular coordinate of r, and R_1 and R_2 are the inside and outside radii of the spring respectively, the relation between the spin induced angle of unwind θ_ω, the torque T_ω and the angular velocity of spin ω is [4]

$$22) \qquad \theta_\omega = \frac{1}{EI} \left\{ T_\omega \ell - \rho R_2^{\,4} \left[1 - 2 \frac{R_2 - R_1}{R_2} + \frac{5}{3} \frac{(R_2 - R_1)^2}{R_2^{\,2}} + \frac{1}{2} \frac{(R_2 - R_1)^3}{R_2^{\,3}} \right] (2n + 1) \pi \omega^2 \right\}$$

where ρ is mass per unit length of spring and θ is angle of unwind. Setting $\theta_\omega = 0$ gives the spin induced torque as a function of the spring geometry

$$23) \qquad T_\omega = \frac{\rho R_2^{\,4}}{\ell} \left[1 - 2 \frac{R_2 - R_1}{R_2} + \frac{5}{3} \frac{(R_2 - R_1)^2}{R_2^{\,2}} + \frac{1}{2} \frac{(R_2 - R_1)^3}{R_2^{\,3}} \right] (2n + 1) \pi \omega^2$$

<u>Spring Turns as a Function of Angle of Unwind</u> The number of spring turns n as a function of θ may be written

24) $$n = n_o - \frac{\theta}{2\pi}$$

where n_o is the initial number of turns.

<u>Spin Decay</u> To a good approximation, spin decay may be taken to be a linear function of θ [5], ie.,

25) $$\omega = \omega_o\ (1 - q\theta)$$

where ω_o is the angular velocity at the muzzle of the gun and q is a constant determined by experiment. Assuming ω decays p percent per turn of the spring arbor, then

26) $$q = \frac{p}{200\pi}$$

and

27) $$\omega = \omega_o\ (1 - \frac{p\theta}{200\pi})$$

or

28) $$\omega/_{\omega_o} = 1 - \frac{p\theta}{200\pi}$$

<u>Segmental Gear Compensator</u> One type of compensator [5] consists of two weighted segmental gears engaging a pinion. Such an arrangement (with both gears represented by a single gear) is shown schematically in Fig. 5. Referring to the figure, the force accelerating the compensator mass toward the center of rotation is

29) $$F = M\,\omega^2 v$$

where M may be considered to be the mass associated with two segmental gears. The moment of F about the center of rotation of the gear is

30) $$T' = M\,\omega^2 v\,(r_p + R_g)\,\text{Sin}\alpha$$

and the reaction tangential force at the rim of the gear is

31) $$F' = \frac{M\,\omega^2 v}{R_g}\,(r_p + R_g)\,\text{Sin}\alpha$$

The opposite of this force acts on the pinion and gives rise to the pinion torque

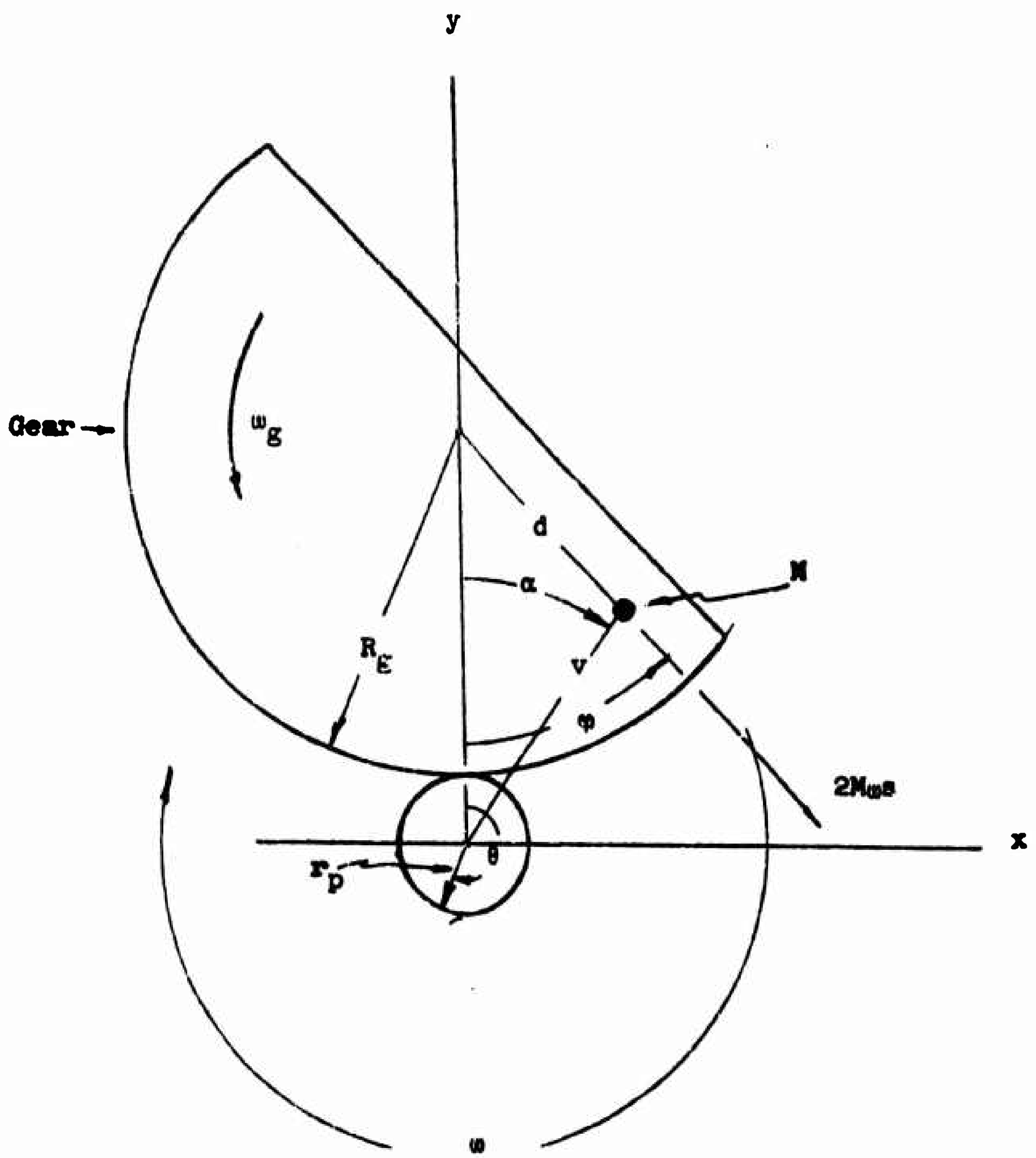


Fig. 5

32) $$T_{c1} = \frac{r_p}{R_g} M \omega^2 v \, (r_p + R_g) \, \mathrm{Sin}\alpha$$

however,

33) $$v \, \mathrm{Sin}\alpha = d \, \mathrm{Sin}\varphi$$

therefore 32) may be written

34) $$T_{c1} = Md \left[\frac{r_p}{R_g} (r_p + R_g) \right] \omega^2 \, \mathrm{Sin}\varphi$$

In addition

35) $$r_p \theta = R_g \varphi$$

and

36) $$T_{c1} = Md \left[\frac{r_p}{R_g} (r_p + R_g) \right] \omega^2 \, \mathrm{Sin} \frac{r_p}{R_g} \theta$$

There will be a Coriolis force $2M\omega s$ acting through the center of rotation of the gear. The speed s in the rotating frame of reference is

37) $$s = \left[\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 \right]^{1/2}$$

where

$$\frac{dx}{dt} = (\omega_g \operatorname{Cos} \omega_g t)d$$

$$\frac{dy}{dt} = (\omega_g \operatorname{Sin} \omega_g t)d$$

and ω_g is the angular velocity of the segmental gear. Assuming ω to be 1000π radians per second and ω_g to be $\pi/50$ radians per second the Coriolis force is

38) $$F_c = \frac{40 \pi^2 w_g d}{386}$$

$$\approx w_g d$$

where w_g is the weight of the gear and mass and d is the distance from the center of rotation of the gear to the center of gravity of the gear and mass. Setting $d = 1.21$ inches and $w_g = .165$ pounds, the Coriolis force is .2 pounds.

<u>Connected Mass Compensator</u> Another type of compensator that could be employed is shown schematically in Fig. 6. The force accelerating the compensator mass toward the center of rotation is

39) $$F = M\omega^2 h$$

From equilibrium, the force transmitted to the arbor is

40) $$P = M\omega^2 h \cos\gamma$$

$$= M\omega^2 L$$

The torque about the center of the arbor due to P is

41) $$T_{c2} = M\omega^2 L r_a$$

which may be written

42) $$T_{c2} = M r_a \, (L_o + r_a \theta)\omega^2$$

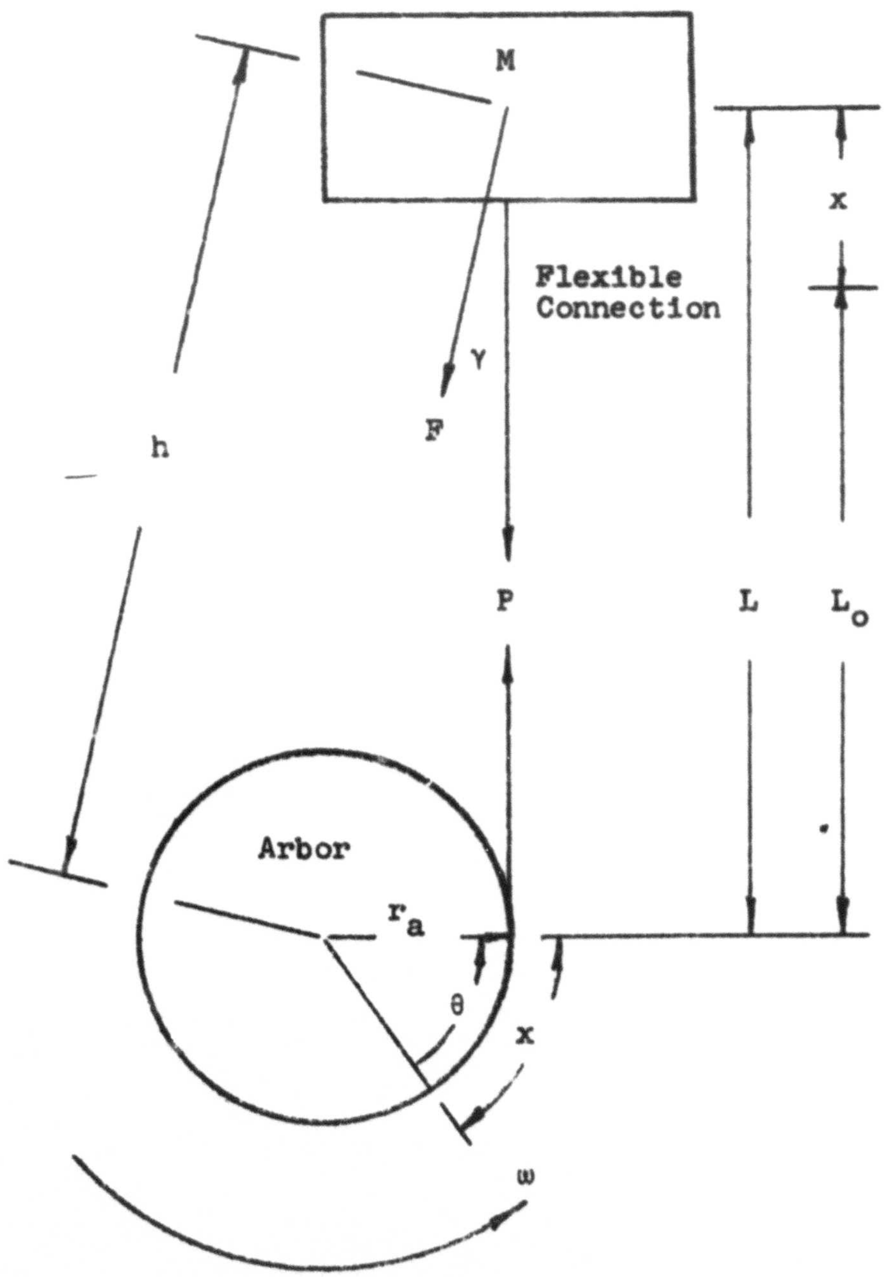


Fig. 6

SYNTHESIS

Symbols

$(T_{f2})_i$	Friction torque between ith and i - 1 leaves
T_{f1}	Torque due to spin induced friction in gear train
T_s	Torque due to windup of mainspring
T_ω	Torque due to spin of mainspring
T_{c1}	Torque due to segmental gear compensator
T_{c2}	Torque due to attached mass compensator
r_a	Radius of mainspring arbor
m_k	Mass of a particular gear-shaft assembly
d_k	Diameter of a particular shaft
e_k	Eccentricity of a particular shaft
g_k	Gear ratio from shaft to main spring
u	Number of gear-shaft assemblies
u_1	Coefficient of friction between shaft and bearing
μ_2	Coefficient of friction between spring leaves
ℓ_i	Length of ith spring turn
t	Thickness of spring
w	Width of spring

ℓ	Length of spring
E	Young's modulus
I	Moment of inertia of cross section of spring
ρ	Mass of spring leave per unit length
R_1	Inside radius of spring
R_2	Outside radius of spring
θ	Angle of unwind of spring arbor
θ_o	Angle of windup of spring arbor
ω	Angular velocity of projectile spin
D	Spring constant (see Fig. 4)
d_n	Distance between spring leaves
n	Number of leaves (turns) of spring
i	Spring leave number counting from arbor
M	Compensator mass
L_o	Initial length of compensator link
r_p	Radius of segmental-gear pinion
R_g	Radius of segmental gear
d	Distance from c.g. of segmental gear to center of rotation of gear

For ease of reference, the relations pertinent to a compensated design are repeated.

a) Gear Train Friction (Relation 6)

$$T_{f1} = \frac{2\mu_1}{\pi} \left[\sum_{j=1}^{u} d_k m_k e_k g_k \right] \omega^2$$

b) Friction Between Spring Leaves (Relation 20)

$$(T_{f2})_i = \frac{120\ \mu_2 tw}{10^4} \left[r_a + (i - 1)t \right] \Big\{ A(n + 1 - i)$$

$$+ \frac{B}{2} \left[n(n + 1) - i(i - 1) \right] + \frac{C}{6} \left[n(n + 1)(2n + 1) \right.$$

$$\left. - i(i - 1)(2i - 1) \right] \Big\} \omega^2$$

c) Windup Torque of Spring (Relation 21)

$$T_s = \frac{EI}{\ell} (\theta_o - \theta) + D$$

d) Torque Due to Spin of Spring (Relation 23)

$$T_\omega = \frac{\rho R_2^4}{\ell} \left[1 - 2 \frac{(R_2 - R_1)}{R_2} + \frac{5}{3} \frac{(R_2 - R_1)^2}{R_2^2} \right.$$

$$\left. + \frac{1}{2} \frac{(R_2 - R_1)^3}{R_2^3} \right] (2n + 1)\pi\omega^2$$

e) Number of Spring Turns as a Function of Angle of Unwind (Relation 24)

$$n = n_o - \frac{\theta}{2\pi}$$

f) Spin Decay (Relation 27)

$$\omega = \omega_o \left(1 - \frac{p\theta}{200\pi}\right)$$

g) Segmental Gear Compensator (Relation 36)

$$T_{c1} = Md \left[\frac{r_p}{R_g} (r_p + R_g) \right] \omega^2 \operatorname{Sin} \frac{r_p}{R_g} \theta$$

h) Connected Mass Compensator (Relation 42)

$$T_{c2} = Mr_a \, (L_o + r_a\theta)\omega^2$$

To synthesize a fully compensated system requires that the above relations be combined to give an expression for torque as a function of θ and that the parameters of the system then be adjusted until the coefficients of θ become zero.

<u>Segmental Gear Compensator</u> Combining a), b), c), d), and g) gives

$$T = \Gamma - \frac{2\mu_1}{\pi} \sum_{k=1}^{u} d_k m_k e_k g_k$$

$$- \frac{120\ \mu_2\ tw}{10^4} [r_a + i - 1)t] \Big\{A(n + 1 - i)$$

$$+ \frac{B}{2} [n(n + 1) - i(i - 1)] + \frac{C}{6} [n(n + 1)(2n + 1)$$

43) $$- i(i - 1)(2i - 1)]\Big\} + \frac{\rho\ R_2^{\ 4}}{l} [1 - 2\ \frac{(R_2 - R_1)}{R_2}$$

$$+ \frac{5}{3} \frac{(R_2 - R_1)^2}{R_2^{\ 2}} + \frac{1}{2} \frac{(R_2 - R_1)^3}{R_2^{\ 3}}]\ (2n + 1)\pi$$

$$+ Md \frac{r_p}{R_g} (r_p + R_g)\ \mathrm{Sin}\ \frac{r_p}{R_g}\ \theta\ \Big]\ \omega^2$$

$$+ \Big[\frac{EI}{l} (\theta_0 - \theta) + D\Big]$$

where

$$A = -\pi(2r_a t - t^2)$$

18) $$B = \frac{\pi}{2} (2r_a d_n - 3td_n - 4t^2)$$

$$C = \frac{\pi}{2} (d_n^2 + 2td_n)$$

and

15) $$d_n = \frac{2 [\ell/\pi - tn^2 - 2r_a n]}{n(n + 1)}$$

Relation f) may be introduced in 43) in order to bring in spin decay effects. Thus using f) and setting

44) $$A' = \frac{2\mu_1}{\pi} \sum_{j=1}^{u} d_k m_k e_k g_k$$

45) $$B' = \frac{120\mu_2 \ tw}{10^4} \ [r_a + (i - 1)t] \left\{A(n + 1 - i)\right.$$

$$+ \frac{B}{2} [n(n + 1) - i(i - 1)] + \frac{C}{6} [n(n + 1(2n + 1)$$

$$\left. - i(i - 1)(2i - 1)]\right\}$$

46) $$C' = \frac{\rho R_2^4}{\ell} \left[1 - 2\frac{(R_2 - R_1)}{R_2} + \frac{5}{3} \frac{(R_2 - R_1)^2}{R_2^2}\right.$$

$$+ \frac{1}{2} \frac{(R_2 - R_1)^3}{R_2^{\,3}} \Big] (2n + 1)\pi$$

47) $$D' = Md \frac{r_p}{R_g} (r_p + R_g) \operatorname{Sin} \frac{r_p}{R_g} \theta$$

and re-grouping gives

$$\left[(C' + D') - (A' + B') \right] \omega_o^{\,2} + \frac{EI}{l} \theta_o + D$$

48) $$- \left\{ [(C' + D') - (A' + B')]\omega_o^{\,2} \frac{2p}{200\pi} + \frac{EI}{l} \right\} \theta$$

$$+ \left\{ [(C' + D') - (A' + B')]\omega_o^{\,2} \left(\frac{p}{200\pi}\right)^2 \right\} \theta^2 = T$$

Use of relation e) in order to bring in rundown effects introduces very great complexity because of B', consequently the synthesis procedure that follows is based upon torque conditions at various specific states of rundown.

Compensation under the conditions implied in 48) can be obtained if the following relations are satisfied

49) $$\left[(C' + D') - (A' + B') \right] \omega_o^{\,2} + \frac{EI}{l} \theta_o + D = T$$

50) $$\left[(C' + D') - (A' + B')\right] \omega_0^2 \frac{2p}{200\pi} + \frac{EI}{l} = 0$$

51) $$\left[(C' + D') - (A' + B')\right] \omega_0^2 \left(\frac{p}{200\pi}\right)^2 = 0$$

Thus if

52) $$C' + D' = A' + B'$$

and

53) $$\frac{EI}{l} \ll T$$

then

54) $$T = \frac{EI}{l} \theta_0 + D$$

which is to say that output torque will remain constant with a magnitude T.

Relation 53) appears to be usually satisfied in standard designs, ie., $EI/l \approx .1$ inch pounds.

It may be seen that A' and B' introduce friction torque losses due to bearing friction and spring friction respectively while C' and D' introduce torque gain due to inertial effects on spring leaves and the compensating mass respectively. It should be pointed out that the physical configuration from which B' and C' are derived are inconsistent inasmuch as B' requires asymmetry whereas C' requires symmetry. It must therefore be assumed that C' is on the order of magnitude of the torque gain for an asymmetrical spring.

It should also be pointed out that 47) is a function of θ and that effective compensation requires that $r_p\theta/R_g$ be limited to the interval in the neighborhood of $\pi/2$. This limitation implies that Sin $\frac{r_p}{R_g}\theta$ has been expanded in a Taylor series about the point $\frac{r_p}{R_g}\theta = \frac{\pi}{2}$ and that all but the first term of the series has been discarded. Other equations similar to 52) can, for example, be developed by expanding Sin $\frac{r_p}{R_g}\theta$ about a point in the interval $0 \leq \frac{r_p}{R_g}\theta \leq \pi/2$ and discarding all but the first or the first two terms. However, the investigation of optimum points about which to expand Sin $\frac{r_p}{R_g}\theta$ should be held in abeyance until more insight is gained into the basic characteristics of the overall device.

<u>**Connected Mass Compensator**</u> **For the connected mass compensator relations a), b), c), d), and h) give**

$$T = \Gamma - 2\mu_1 \sum_{k=1}^{u} d_k m_k e_k g_k - \frac{120\ \mu_2\ tw}{10^4}$$

$$[r + (i - 1)t]\Big\{A(n + 1 - i)$$

$$+ \frac{B}{2}[n(n + 1) - i(i - 1)] + \frac{C}{6}[n(n + 1)(2n + 1)$$

54) $$- i(i - 1)(2i - 1)]\Big\} + \frac{\rho\ R_2^{\ 4}}{\ell}[1 - 2\frac{(R_2 - R_1)}{R_2}$$

$$+ \frac{5}{3}\frac{(R_2 - R_1)^2}{R_2^{\ 2}} + \frac{1}{2}\frac{(R_2 - R_1)^3}{R_2^{\ 3}}](2n + 1)\pi$$

$$+ Mr_a\ (L_o + r_a\theta)\Big]\omega^2$$

$$+ \Gamma\frac{EI}{\ell}(\theta_o - \theta) + D\Big]$$

From f), 44), 45), 46), and 54)

$$T = \Big\{[C' - (A' + B') + Mr_a\ L_o]\ \omega_o^{\ 2} + \frac{EI}{\ell}\ \theta_o + D\Big\}$$

$$- \left\{ [C' - (A' + B') + Mr_a L_o] \frac{2p}{200\pi} \omega_o^2 \right.$$

55) $$\left. - Mr_a^2 \omega_o^2 - \frac{EI}{l} \right\} \theta$$

$$+ \left\{ [C' - (A' + B') + Mr_a L_o] \left(\frac{p}{200\pi}\right)^2 \right.$$

$$\left. - Mr_a^2 \frac{2p}{200\pi} \right\} \omega_o^2 \theta^2$$

$$+ \left\{ Mr_a^2 \left(\frac{p}{200\pi}\right)^2 \omega_o^2 \right\} \theta^3$$

Compensation under the conditions implied in 55) can be obtained if the following relations are satisfied

56) $$[C' - (A' + B') + Mr_a L_o] \omega_o^2 + \frac{EI}{l} \theta_o + D = T$$

57) $$[C' - (A' + B') + Mr_a L_o] \frac{2p}{200\pi} \omega_o^2$$

$$- Mr_a^2 \omega_o^2 + \frac{EI}{l} = 0$$

58) $$[C' - (A' + B') + Mr_a L_o] \left(\frac{p}{200\pi}\right)^2$$

$$- Mr_a^2 \frac{2p}{200\pi} = 0$$

59) $$Mr_a^2 \left(\frac{p}{200\pi}\right)^2 \omega_o^2 = 0$$

If

60) $$C' + Mr_a L_o = A_1 + B'$$

then from 56)

61) $$\frac{EI}{l} \theta_o + D = T$$

and from 57)

62) $$- Mr_a^2 \omega_o^2 + \frac{EI}{l} = 0$$

and from 58)

63) $$Mr_a^2 \frac{2p}{200\pi} = 0$$

From 61) and 62)

64) $$T = Mr_a^2 \omega^2 \theta_o + D$$

but 59) and 63) cannot be satisfied unless spin decay is zero. Thus the design equation 60) will give good results only if

65) $$Mr_a^2 \left(\frac{p}{200\pi}\right)^2 \omega_o^2 << T$$

and

66) $$Mr_a^2 \left(\frac{2p}{200\pi}\right) << T$$

Dividing 63) by 59) gives

$$\frac{Mr_a^2 \left(\frac{2p}{200\pi}\right)}{Mr_a^2 \left(\frac{p}{200\pi}\right)^2 \omega_o^2} = \frac{400\pi}{p\omega_o^2}$$

Assuming $\omega_o \geq 10$ and $p \geq 1$, 63) will be larger than 59) and it is only necessary to satisfy 66).

EXAMPLES

Gear Train Friction The gear train friction torque reflected to the main spring may be calculated from relation 6)

$$6) \qquad T_{f1} = \left[\frac{2\mu_1}{\pi} \sum_{k=1}^{u} d_k m_k e_k g_k\right] \omega^2$$

where μ_1 is coefficient of friction, u is the number of gear-shaft assemblies, d_k is the diameter of a particular shaft, m_k is the mass of a particular shaft-gear assembly, e_k is the eccentricity (distance from center of shell rotation) of a particular shaft, and g_k is the gear ratio from a particular shaft to the mainspring. The bracketed term in 6) has been calculated using the values shown in Table 1 and gives

$$67) \qquad T_{f1} = 3.95\ (10^{-6})\ \omega^2 \qquad \text{in.oz.}$$

$$= A'\ \omega^2$$

Mainspring Friction The torque due to friction between the i'th and i - 1 spring leaves may be calculated from 20)

$$(T_{f2})_i = 120\mu_2\, tw(10^{-4}) \left[r_a + (i - 1)t\right]\Big\{A(n + 1 - i)$$

20) $$+ \frac{B}{2}\left[n(n + 1) - i(i - 1)\right] + \frac{C}{6}\left[n(n + 1)(2n + 1)\right.$$

$$\left.\left. - i(i - 1)(2i - 1)\right]\Big\}\, \omega^2$$

where

$$A = -\pi(2r_a t - t^2)$$

$$B = \frac{\pi}{2}\,(2r_a\, d_n - 3td_n - 4t^2)$$

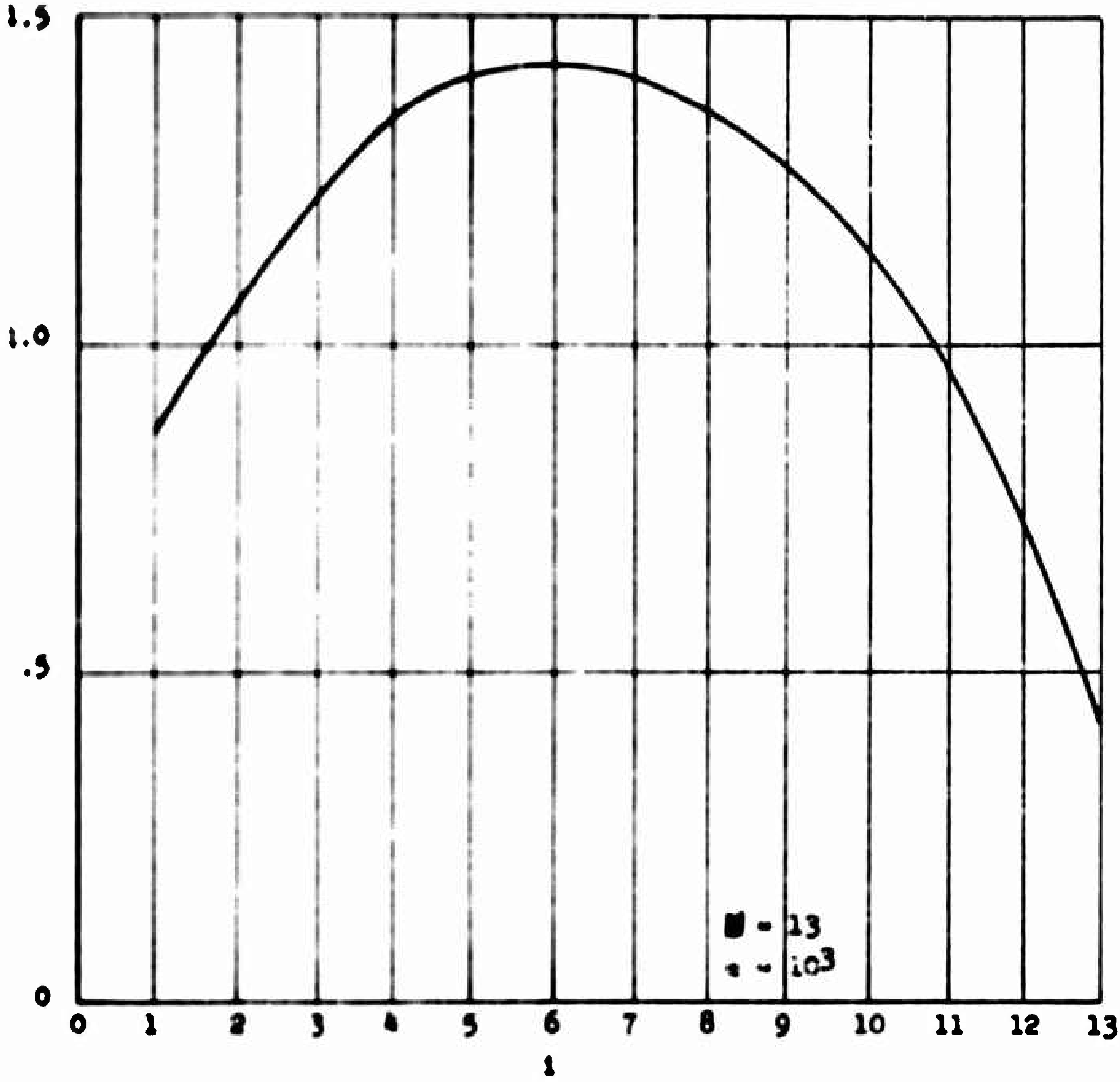
$$C = \frac{\pi}{2}\,(d_n^{\,2} + 2td_n)$$

$$d_n = \frac{2[l/\pi - tn^2 - 2r_a n]}{n(n + 1)}$$

The variation of $(T_{f2})_i$ with respect to i is shown on page 36 for the following quantities

$\mu_2 = .17$ coefficient of friction

$r_a = .14''$ radius of arbor



SPIN INDUCED FRICTION TORQUE

BETWEEN ITH AND I-1 LEAVES OF

SHIMS

TABLE 1

Gear Train Friction

Mechanism in Center

$\mu_1 = .17$

k (Shaft) no.	d_k (inches)	m_k (oz.sec.^{2}/in.)	e_k (inches)	g_k	$\frac{2\mu_1}{\pi} d_k m_k e_k g_k$
us	$97.8(10^{-3})$	$228(10^{-6})$	0	1	0
4	$49.2(10^{-3})$	$134(10^{-6})$	.314	2.33	$521(10^{-9})$
3	$30.0(10^{-3})$	$40.3(10^{-6})$	.284	7.0	$270(10^{-9})$
2	$22.1(10^{-3})$	$25.6(10^{-6})$	.312	21.10	$401(10^{-9})$
1	$18.3(10^{-3})$	$20.9(10^{-6})$	.210	78.8	$684(10^{-9})$
0	$18.6(10^{-3})$	$13.8(10^{-6})$	.211	354	$2070(10^{-9})$
BL	$18.0(10^{-3})$	$7.69(10^{-6})$	0	-	-

$$\frac{2\mu_1}{\pi} \sum_{k=1}^{u} d_k m_k e_k g_k = 3946(10^{-9})$$

$w = .25''$ width of spring

$t = .015''$ thickness of spring

$\ell = 26.5''$ length of spring

$n = 13$ number of turns of spring
($n = 16.2$ when $d_n = 0$)

$\omega = 1000$ angular velocity of spin

The shape of the torque versus spring-leave curve suggests that relative motion between spring leaves begins at the outside of the spring where the friction torque is the least and that gross readjustments may occur as the spring unwinds.

For $n = 13$, the maximum friction torque between spring leaves is, from the above theory

$$(68) \qquad (T_{f2})_6 = 1.425(10^{-6})\ \omega^2$$

$$= B'\ \omega^2 \qquad \text{in.oz.}$$

<u>Mainspring Torque</u> If the length, width, and thickness of the mainspring are taken as above and if E is $48(10^{-7})$ ounces per square inch, then EI/ℓ is 1.28 inch ounces and [relation 21)]

69) $$T_s = .0797\ (\theta_0 - \theta) + D$$

The torque gain due to the spin of the spring may be calculated from 23)

23) $$T_\omega = \frac{\rho R_2^4}{\iota}\left[1 - 2\frac{(R_2 - R_1)}{R_2} + \frac{5}{3}\frac{(R_2 - R_1)^2}{R_2^2} + \frac{1}{2}\frac{(R_2 - R_1)^3}{R_2^3}\right](2n + 1)\,\pi\,\omega^2$$

If R_1 and R_2, the inside and outside radii of the spring, are taken to be .14" and .6" respectively, and if n and ι are taken as above then 23) becomes

70) $$T_\omega = 9(10^{-6})\ \omega^2 \qquad \text{in.oz.}$$

$$= C'\ \omega^2$$

<u>Segmental Gear Compensator</u> The torque gain due to the segmental compensator is [relation 36]

36) $$T_{c1} = Md\left[\frac{r_p}{R_g}(r_p + R_g)\ \omega^2 \sin\frac{r_p}{R_g}\theta\right]$$

If Sin $\frac{r_p}{R_g}$ θ is approximated by the first term of a Taylor's expansion about θ $\frac{\pi R_g}{r_p}$ then 36) becomes

$$71) \qquad T_{c1} = Md\left[\frac{r_p}{R_g}(r_p + R_g)\right]\omega^2$$

$$= D'\ \omega^2$$

If the appropriate parts of 67), 68), 70), and 71) are substituted in 52), then for n = 13 the design criterion for a segmental gear compensator becomes

$$72) \qquad 9(10^{-6}) + Md\left[\frac{r_p}{R_g}(r_p + R_g)\right] = 3.95(10^{-6}) + 1.425(10^{-6})$$

where the terms in 72) may be identified from

$$52) \qquad C' + D' \quad = \quad A' + B'$$

It thus appears that C', the term contributed by the torque gain due to the spinning spring, is by itself sufficiently large to compensate for A' and B' which are the terms contributed by torque losses due to gear train and spring friction respectively. However, it should be remembered that C' strongly depends upon n, R_2, and the analytical function chosen to represent the spring. It would seem to be highly desirable to investigate this term more thoroughly.

If C' is neglected then 72) becomes

$$\text{73)} \qquad Md \left[\frac{r_p}{R_g} (r_p + R_g) \right] = 5.375(10^{-6})$$

and if [5]

$$r_p + R_g = .452$$

$$r_p = \frac{1}{3} R_g$$

$$d = .19$$

then 73) gives $M = 1.875(10^{-4})$ which corresponds to a weight of .0725 oz., a feasible order of magnitude.

Connected Mass Compensator If the appropriate parts of 67), 68) and 70) are substituted in 60) then for n = 12 the design criterion for a connected mass compensator becomes

$$\text{74)} \qquad 9(10^{-6}) + Mr_a L_o = 3.95(10^{-6}) + 1.425(10^{-6})$$

where the terms in 74) may be identified from

60) $$C' + Mr_a L_o = A' + B'$$

Inasmuch as the remarks made earlier about C' also apply to 74) it will be neglected. Relation 74) then becomes

75) $$Mr_a L_o = 5.375(10^{-6})$$

If r_a and L_o are taken to be .14" and .2" respectively then $M = 1.92(10^{-4})$ which corresponds to a weight of .0741 ounces, a feasible order of magnitude.

Relation 66) gives

76) $$\frac{4.62}{10^6} p << T$$

which is satisfied for any conceivable value of p. Thus 74) is valid.

CONCLUSIONS

It would seem that inertially generated torques can be used to compensate for friction, run down, and spin decay energy losses in timing mechanisms. However, it appears that two elements of the timing mechanism should be investigated further before dependable design procedures can be developed. These areas are the torque gain due to spring spin, and torque loss due to friction between spring leaves.

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The behavior of timing mechanisms under spin conditions is affected by: 1) changes in gear train friction, 2) changes in friction between main spring leaves, 3) changes in mainspring torque. The purpose of this study has been to examine these effects in the light of the possibility of synthesizing an inertial (spin energized) compensator which will provide enough torque to balance out both spin induced and run-down torque losses.

The problem has been approached by analyzing various sources of torque loss or gain and then combining these analyses to show the pertinent design parameters.

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